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RADIO WAVE PROPAGATION IN STRUCTURED IONIZATION FOR SATELLITE APPLICATIONS II

Defense Nuclear Agency
Atmospheric Effects Division
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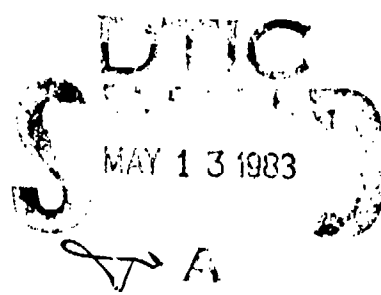
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This report is an extension of "Radio Wave Propagation in Structured Ionization for Satellite Applications," DNA 5304D, Dec 79. That document presented algorithms for calculating signal structure parameters which characterize scintillated radio signals. This report extends the applicability of that work to include locally homogeneous spectral indices less than two. The report also describes the phase effects only component of a scintillated signal and folds antenna effects into the formalism. Finally, a number of loose ends from DNA 5304D were treated.

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PREFACE

The models and algorithms in this paper represent a synthesis of results from different works by several different individuals over the past few years. In particular, I would like to acknowledge the contributions of the following people: Dr. Walter Chesnut and Dr. Charles Rino of SRI International, Dr. K. C. Yeh and his colleagues at the University of Illinois at Urbana, Dr. David Sachs of Science Applications Incorporated, Dr. Roy Hendrick, Mr. Robert Bogusch, Dr. Dennis Knepp of Mission Research Corporation, Dr. Clifford Prettie of Berkeley Research Associates, and Dr. E. J. Fremouw of Physical Dynamics Incorporated.

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RADIO WAVE PROPAGATION IN STRUCTURED IONIZATION
FOR SATELLITE APPLICATIONS II

1. INTRODUCTION

This report is an extension of reference 1 which developed the
algorithms necessary to calculate the signal structure parameters and the
generalized power spectrum which represent the simultaneous phase and
amplitude effects of propagating electromagnetic waves through media
characterized by a structured index of refraction. Portions of that
work were restricted to structure spectral indices greater than or equal
to two. Also, the effects description did not cover the phase effects
only component of the signal. This paper will reduce these restrictions.

2. ENVIRONMENT CHARACTERIZATION

The first step in any propagation study is to characterize the
ionization or equivalently the index of refraction fluctuations of the
ionospheric propagation environment. Figure 1 illustrates the geometry of
a typical satellite link. The index of refraction fluctuations, represented
schematically by the curved lines are typically elongated along the magnetic
field. $\hat{t}$ is a unit vector along the magnetic field and is a slowly varying
function of space since the field lines are curved. The z axis and the
$\hat{z}$ unit vector represent the propagation line of sight. The transmitter
by definition, is at $z=0$ and the receiver is at $z=z_t$. The $\hat{r}$ and $\hat{s}$
unit vectors complete with $\hat{t}$ an orthogonal coordinate system that is used
to define the structure. The orientation of $\hat{r}$ and $\hat{s}$ is chosen to
best represent any anisotropy of the index of refraction structure
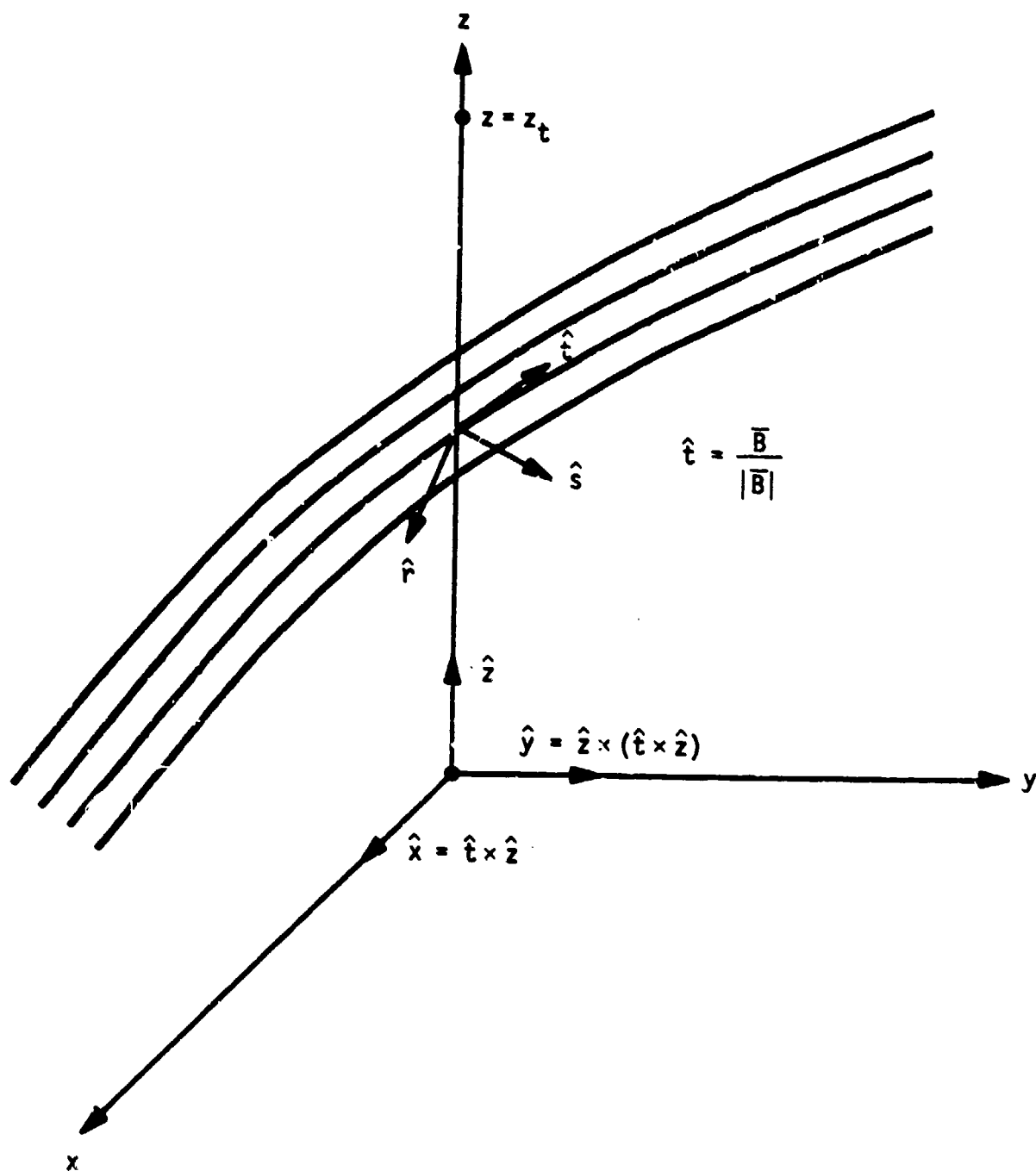about the field line. As with $\hat{t}$, $\hat{r}$ and $\hat{s}$ may be slow functions of

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Figure 1. Propagation geometry.

position. In the $\hat{r}$, $\hat{s}$, and t system the index of refraction fluctuations are often represented by a power law power spectral density.

$$PSD(K_r,K_s,K_t) = \frac{8\pi^{3/2}\overline{\Delta n_i^2}L_rL_sL_t\Gamma(n)/\Gamma(n-3/2)}{(1+L_r^2K_r^2+L_s^2K_s^2+L_t^2K_t^2)^n} \qquad (1)$$

where

K_r,K_s,K_t = spatial wave numbers

L_r,L_s,L_t = structure outer scales

$\overline{\Delta n_i^2}$ = index of refraction variance

$\Gamma(n)$ = gamma function of argument n

$2n - 2$ = fluctuation spectral index

Both in situ measurements [2-9] and theoretical considerations [10] imply that $n\approx2$ for ionospheric structured environments. This spectral slope is believed to be caused by the existence of very sharp edged structures. The fluctuation power spectrum is considered a locally homogeneous quantity as discussed by Tatarski [11]. This means that the parameters can be slowly varying functions in any space direction with respect to the spatial scale in that direction.

The structure variation of the index of refraction perpendicular to the $\hat{z}$ axis dominates the propagation effects while the variation parallel to the $\hat{z}$ axis enters only through the strength of the integrated phase variance. Thus Equation 1 must be transformed to a frame with one axis being the $\hat{z}$ axis. These new axes are defined by:

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$$\hat{x} = \hat{t} \times \hat{z} / |\hat{t} \times \hat{z}| \tag{2a}$$

$$\hat{y} = \hat{z} \times (\hat{t} \times \hat{z}) / |\hat{z} \times (\hat{t} \times \hat{z})| \tag{2b}$$

Two rotations suffice to accomplish the transformation. First, the $\hat{r}$ axis is rotated about the $\hat{t}$ axis by angle ϕ to become parallel to the $\hat{x}$ axis. ϕ is defined by

$$\hat{t}\sin\phi = \hat{r} \times (\hat{t} \times \hat{z}) \tag{3a}$$

$$\cos\phi = \hat{r} \cdot (\hat{t} \times \hat{z}) \tag{3b}$$

Next, the $\hat{t}$ axis is rotated about the $\hat{x}$ axis by angle Φ into the $\hat{z}$ axis. Φ is defined by

$$\hat{x}\sin\Phi = \hat{t} \times \hat{z} \tag{4a}$$

$$\cos\Phi = \hat{t} \cdot \hat{z} \tag{4b}$$

These transformations can be simplified by defining new effective scale sizes.

$$L_x^2 = L_r^2\cos^2\phi + L_s^2\sin^2\phi \tag{5a}$$

$$L_y^2 = (L_r^2\sin^2\phi + L_s^2\cos^2\phi)\cos^2\Phi + L_t^2\sin^2\Phi \tag{5b}$$

$$L_z^2 = (L_r^2\sin^2\phi + L_s^2\cos^2\phi)\sin^2\Phi + L_t^2\cos^2\Phi \tag{5c}$$

$$L_{xy} = (L_s^2 - L_r^2)\cos\Phi\cos\phi\sin\phi \tag{5d}$$

$$L_{xz} = (L_r^2 - L_s^2)\sin\Phi\cos\phi\sin\phi \tag{5e}$$

$$L_{yz} = (L_t^2 - L_r^2\sin^2\phi - L_s\cos^2\phi)\cos\Phi\sin\Phi \tag{5f}$$

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The final result for the locally homogeneous index of refraction power spectral density is

$$P(K_x, K_y, K_z) = \frac{8\pi^{3/2}\overline{\Delta n_i^2}L_r L_s L_t \Gamma(n)/\Gamma(n-3/2)}{(1+L_x^2 K_x^2 + L_y^2 K_y^2 + L_z^2 K_z^2 + 2L_{xy}K_x K_y + 2L_{xz}K_x K_z + 2L_{yz}K_y K_z)^n}$$

(6)

3. DERIVATION OF SIGNAL STRUCTURE PARAMETERS

The required signal parameters are derived from the differential phase spectrum which is Equation 6 with K_z set to zero and multiplied by K^2, where K is the carrier frequency wave number. The differential phase spectrum is

$$\frac{dP_\phi(K_x, K_y)}{dz} = \frac{8\pi^{3/2}K^2\overline{\Delta n_i^2}L_r L_s L_t \Gamma(n)/\Gamma(n-3/2)}{(1+L_x^2 K_x^2 + L_y^2 K_y^2 + 2L_{xy}K_x K_y)^n}$$

(7)

The mean square integrated phase fluctuation is

$$\sigma_\phi^2 = \int_0^{z_t} dz \; \frac{d\sigma_\phi^2}{dz}$$

(8)

where

$$\frac{d\sigma_\phi^2}{dz} = \frac{2\pi^{\frac{1}{2}}\Gamma(n-1)}{\Gamma(n-3/2)} \; K^2\overline{\Delta n_i^2}L_z$$

(9)

$$L_z = \frac{L_r L_s L_t}{(L_x^2 L_y^2 - L_{xy}^2)^{\frac{1}{2}}} \tag{10}$$

L_z is redefined as the effective $\hat{z}$ axis scale. Finally

$$\frac{dP_\phi(K_x, K_y)}{dz} = \frac{d\sigma_\phi^2}{dz} \frac{4\pi (L_x^2 L_y^2 - L_{xy}^2)^{\frac{1}{2}} (n-1)}{(1 + L_x^2 K_x^2 + L_y^2 K_y^2 + 2L_{xy} K_x K_y)^n} \tag{11}$$

Equation 9 requires n greater than 1.5 which is an artifact of the spectral form chosen for the index of refraction fluctuations. Equation 11 requires n greater than one if $d\sigma_\phi^2/dz$ is suitably redefined consistent with Equation 8. The following development applies to $1.2 \leq n \leq 4$ where a well defined $d\sigma_\phi^2/dz$ is assumed.

Equation 11 can be Fourier transformed to

$$\frac{dR_\phi(x,y)}{dz} = \frac{d\sigma_\phi^2}{dz} \frac{\rho^{n-1} K_{n-1}(\rho)}{2^{n-2} \Gamma(n-1)} \tag{12a}$$

$$\cong \frac{d\sigma_\phi^2}{dz} \left(1 - B_n(\rho^2)^{m/2} + \cdots \right. \tag{12b}$$

where

$$\rho^2 = \frac{L_x^2 y^2 - 2L_{xy} xy + L_y^2 x^2}{L_x^2 L_y^2 - L_{xy}^2}$$

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$$m = 2, \quad n \geq 2$$

$$m = 2n-2, \quad 1.2 \leq n \leq 2$$

$$B_n = \text{constant of order unity (see Appendix B)}$$

Equations 8, 11, and 12 are the fundamental equations from which all of the required propagation quantities can be derived.

The mean square log amplitude fluctuation is developed in Appendix A where the range of application has been extended from that in Reference 1.

$$\overline{x^2} \simeq \frac{1}{2} \int_0^{z_t} dz \ \frac{d\sigma_\phi^2}{dz} \ (n-1)f(n) \left\{ \frac{8a_1^6}{3M(z)} + \frac{8(a_2^2-c_2^2)c_2^4}{M(z)} \right.$$

$$\left. + \ I_0(z) + \frac{16c_1^{2n}c_2^4 M^{n-1}(z)a_2^{2-2n}}{2n-2} \right\} \tag{13}$$

where
$$I_0(z) = \frac{16c_1^{2n}M^{n-1}(z)(c_2^{6-2n}-a_1^{6-2n})}{6-2n} \ , \quad n \neq 3$$

$$I_0(z) = 16c_1^{2n}M^{n-1}(z)\ln\left\{\frac{c_2}{a_1}\right\}, \quad n=3$$

$$M(z) = \frac{(L_x^2+L_y^2)(z_t-z)z}{Kz_t(L_x^2L_y^2-L_{xy}^2)}$$

$$a_1 = \min\left(c_2, c_1 M^{\frac{1}{2}}(z)\right)$$

$$a_2 = \max\left(c_2, c_1 M^{\frac{1}{2}}(z)\right)$$

$$c_1 = 0.5$$

$$c_2 = 0.841$$

$$f(n) = \left(\frac{f'(n) + M(z)/n}{1 + M(z)}\right)\left\{1 - \frac{n}{12}\exp\left[-\left(\frac{M(z)}{3} - \frac{1}{10M(z)} - 1\right)^2\right]\right\}$$

$$f'(n) = 1.1 - \max\left[0.0, 0.5(n - 2.4)\right]$$

The perpendicular signal decorrelation as developed in Reference 1 already applies over the required range of spectral indices.

$$\tau_\perp = B(n, \sigma_\phi^2)\left[\int_0^{z_t} dz \frac{d\sigma_\phi^2}{dz} B_n\left(\frac{z^2 \rho_v^2}{z_t^2}\right)^{m/2}\right]^{-1/m} \tag{14}$$

where
$$\rho_v^2 = \frac{L_y^2 V_x^2 - 2L_{xy} V_x V_y + L_x^2 V_y^2}{L_x^2 L_y^2 - L_{xy}^2}$$

$$V_x = \hat{x} \cdot \overline{V}$$

$$V_y = \hat{y} \cdot \overline{V}$$

$$\overline{V} = -\left(\frac{z_t - z}{z}\right)\overline{V}_{tr} + \frac{z_t}{z}\overline{V}_{st} - \overline{V}_{re}$$

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$$\overline{V}_{tr} = \text{transmitter velocity}$$

$$\overline{V}_{st} = \text{index of refraction structure velocity}$$

$$\overline{V}_{re} = \text{receiver velocity}$$

$$B(n,\sigma_\phi^2) = \text{minimum}\left[1, (-0.34n^2+2.51n-2.00)(\sigma_\phi^2 B_n)^{1/m}\right]$$

The remaining signal structure parameters are derivable from the mutual coherence function of $U(z,\overline{r},K)$ where the propagating carrier signal is expressed as

$$E(z,\overline{r},K) = \frac{U(z,\overline{r},K)e^{iK(|\overline{r}|^2+z^2)^{\frac{1}{2}}}}{z} \tag{15}$$

The mutual coherence function for U is

$$G(z,x,y,n) = \overline{U^*(z,\overline{r}_1,K_1)U(z,\overline{r}_2,K_2)} \tag{16}$$

where

$$n = (K_2 - K_1)/K_2$$

$$x\hat{x} + y\hat{y} = \overline{r}_2 - \overline{r}_1$$

The equation for G is

$$\frac{dG}{dz} = -\frac{i}{2K_2}\left(\frac{n}{1-n}\right)\left(\frac{d^2G}{dx^2} + \frac{d^2G}{dy^2}\right) - \frac{x}{z}\frac{dG}{dx} - \frac{y}{z}\frac{dG}{dy}$$
$$- \left\{\frac{1}{2}\left[1 + \frac{1}{(1-n)^2}\right]\frac{d\sigma_\phi^2}{dz} - \frac{dR_\phi(x,y)}{dz}\right\}G \tag{17}$$

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First, an approximate parallel signal decorrelation time and angle of arrival distribution will be developed. Let η be zero. Also, let us define a set of fixed axis, $\hat{u}$ and $\hat{v}$, perpendicular to the $\hat{z}$ axis such that $\hat{u} \times \hat{v} = \hat{z}$. At each point on that $\hat{z}$ axis, the rotation angle between the $\hat{x}$ and the $\hat{u}$ axis is defined by

$$\hat{z} \sin\theta = \hat{x} \times \hat{u}$$

$$\cos\theta = \hat{x} \cdot \hat{u}$$

Let

$$L_u^2 = L_y^2 \cos^2\theta + L_x^2 \sin^2\theta - 2L_{xy} \sin\theta\cos\theta$$

$$L_v^2 = L_x^2 \cos^2\theta + L_y^2 \sin^2\theta + 2L_{xy} \sin\theta\cos\theta$$

$$L_{uv} = (L_y^2 - L_x^2) \sin\theta\cos\theta - L_{xy}(\sin^2\theta - \cos^2\theta)$$

The formal solution to Equation 17 for $\sigma_\phi^2 > 1$ is

$$G(z_t, u, v) = e^{-\int_0^{z_t} dz \; \frac{d\sigma_\phi^2}{dz} \; B_n \left[\left(\frac{z}{z_t}\right)^2 \left(\frac{L_u^2 u^2 - 2L_{uv} uv + L_v^2 v^2}{L_x^2 L_y^2 - L_{xy}^2}\right) \right]^{m/2}} \quad (18)$$

The exponential is not factorable so we approximate Equation 18 by

$$G(z_t, u, v) \approx e^{-(C_u u^2 + C_v v^2 - 2C_{uv} uv)^{m/2}} \quad (19)$$

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where

$$C_u = \left\{ \int_0^{z_t} \frac{d\sigma_\phi^2}{dz} B_n \left[\left(\frac{z}{z_t}\right)^2 \frac{L_u^2}{L_x^2 L_y^2 - L_{xy}^2} \right]^{m/2} dz \right\}^{2/m} \qquad (20a)$$

$$C_v = \left\{ \int_0^{z_t} \frac{d\sigma_\phi^2}{dz} B_n \left[\left(\frac{z}{z_t}\right)^2 \frac{L_v^2}{L_x^2 L_y^2 - L_{xy}^2} \right]^{m/2} dz \right\}^{2/m} \qquad (20b)$$

$$C_{uv} = \text{sign}(C'_{uv}) \, |C'_{uv}|^{2/m} \qquad (20c)$$

$$C'_{uv} = \int_0^{z_t} \frac{d\sigma_\phi^2}{dz} B_n \left[\left(\frac{z}{z_t}\right)^2 \frac{|L_{uv}|}{L_x^2 L_y^2 - L_{xy}^2} \right]^{m/2} \text{sign}(L_{uv}) dz \qquad (20d)$$

Equations 19 and 20 are approximate generalizations of Equations 25 and 26 of Reference 1. Equations 19 and 20 agree with Equation 18 when n is greater than or equal to two or if L_u, L_v, and L_{uv} are proportional to each other along the $\hat{z}$ axis. Thus, Equations 19 and 20 are always reasonable approximations for satellite C^3 applications independent of n.

Next, let us rotate the u-v coordinates to remove the cross term in G. Let

$$\tan(2\epsilon) = \frac{2C_{uv}}{C_u - C_v}$$

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$$p = u \cos \epsilon - v \sin \epsilon$$

$$q = u \sin \epsilon + v \cos \epsilon$$

$$C_p = \frac{1}{2} \left[C_u + C_v + \left((C_u - C_v)^2 + 4C_{uv} \right)^{1/2} \right]$$

$$C_q = \frac{1}{2} \left[C_u + C_v - \left((C_u - C_v)^2 + 4C_{uv} \right)^{1/2} \right]$$

Then

$$G(z_t, p, q) = e^{-(C_p F^2 + C_q q^2)^{m/2}} \tag{21}$$

The parallel signal decorrelation time is defined as

$$\tau_\parallel = \frac{3.7K}{(C_p^{2/3} + C_q^{2/3})^{3/2} |(\bar{V}_{re} - \bar{V}_{st}) \cdot \hat{z}|} \tag{22}$$

The final decorrelation time is

$$\tau_o = \text{minimum} \ (\tau_\perp, \tau_\parallel) \tag{23}$$

The angle of arrival distribution is approximated by

$$P_\theta(\theta_p, \theta_q) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} dq \ e^{-(C_p p^2 + C_q q^2)^{m/2}} e^{i\theta_p Kp} e^{i\theta_q Kq} \tag{24}$$

where

$$\int_{-\infty}^{\infty} d\theta_p \int_{-\infty}^{\infty} d\theta_q \; P_\theta(\theta_p, \theta_q) = 1 \qquad (25)$$

When m equals two, Equation 24 is equal to Equation 36 in Reference 1. The measures of the angular scatter are defined by

$$\sigma_{\theta_p}^2 = \frac{2C_p}{K^2} \qquad (26a)$$

$$\sigma_{\theta_q}^2 = \frac{2C_q}{K^2} \qquad (26b)$$

For m equal to two, Equation 26 is the mean square scattering angle. For m less than two, this interpretation is not strictly valid. Nevertheless, $\sigma_{\theta_p}^2$ and $\sigma_{\theta_q}^2$ remain good measures of angular scatter.

The generalized power spectrum can be derived by solving Equation 17 using Equation 12 and the delta layer approximation.

$$\frac{d\sigma_\phi^2}{dz} = \delta(z-z_1) \int_0^{z_t} \frac{d\sigma_\phi^2}{dz} \; dz \qquad (27)$$

To account for time decorrelation, the argument for Equation 12b is modified by incorporating the effective velocity, V_x, and a time displacement argument

$$\rho^2 = \frac{L_x^2 y^2 + L_y^2 (x - V_x \Delta t)^2}{L_x^2 L_y^2} \qquad (28)$$

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The cross term has been eliminated in anticipation of the two final cases: one dimensional structure perpendicular to the line of sight and isotropic structure around the line of sight. The effective velocity is assumed parallel to the x axis. After some algebra we get

$$
\Gamma_2(\theta_x, \theta_y, f, \tau) = \frac{f_2' \tau_0}{2^{\frac{1}{2}} \pi^{3/2} \sigma_\theta^2} \left(\frac{\sigma_\phi R f_2'}{f_c} \right)^{-1} \int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty} dr \; \exp \left\{ -(s^2 + r^2)^{m/2} \right.
$$

$$
\left. -12^{\frac{1}{2}} \frac{s\theta_x}{\sigma_\theta} - 12^{\frac{1}{2}} \frac{r\theta_y}{\sigma_\theta} - \frac{1}{2} \left(\frac{\sigma_\phi R f_2'}{f_c} \right)^{-2} \left[2\pi f_2' \tau \right. \right.
$$

$$
\left. \left. - \frac{\theta_x^2 + \theta_y^2}{2\sigma_\theta^2} \right]^2 \right\} \delta \left(\tau_0 f - \frac{\theta_x}{2^{\frac{1}{2}} \pi \sigma_\theta} \right) \qquad (29a)
$$

for the isotropic structure statistics. For one dimensional structure

$$
\Gamma_1(\theta_x, \theta_y, f, \tau) = \frac{f_1' \tau_0}{\pi^{\frac{1}{2}} \theta} \left(\frac{\sigma_\phi R f_1'}{f_c} \right)^{-1} \int_{-\infty}^{\infty} ds \; \exp \left\{ -(s^2)^{m/2} - 12^{\frac{1}{2}} \frac{s\theta_x}{\sigma_\theta} \right.
$$

$$
\left. - \frac{1}{2} \left(\frac{\sigma_\phi R f_1'}{f_c} \right)^{-2} \left[2\pi f_1' \tau - \frac{\theta_x^2}{2\sigma_\theta^2} \right]^2 \right\} \delta \left(\tau_0 f - \frac{\theta_x}{2^{\frac{1}{2}} \pi \sigma_\theta} \right) \delta(\theta_y) \quad (29b)
$$

where $\sigma_{\phi R}^2$ = Rayleigh phase variance (see Appendix A)

f = scintillation frequency spread

τ = delay

$$f_2' = f_1' = \left[\frac{4\pi(\sigma_\phi B_n)^{2/m}(z_t - z_1)z_1}{K^2 c L_x^2 z_t} \right]^{-1}$$

θ_x, θ_y = energy arrival angle coordinates

The final generalized power spectrum is calculated by integrating the product of Equation 29 and the transmitter and receiver antenna gain functions over the energy angle of arrival coordinates.

$$\Gamma_n(f, \tau) = \int_{-\infty}^{\infty} d\theta_x \int_{-\infty}^{\infty} d\theta_y \Gamma_n(\theta_x, \theta_y, f, \tau) G_R(\theta_x, \theta_y) G_\tau \left[\frac{\sigma_\theta'}{\sigma_\theta} \theta_x, \frac{\sigma_\theta'}{\sigma_\theta} \theta_y \right] \quad (30)$$

where σ_θ' is the root mean square energy angle of arrival with the transmitter and receiver interchanged. Calculations shows that the functional form in Equation 30 works well for thick layers as long as τ_0, f_0, σ_θ, and σ_θ' are chosen properly. Appendix D derives a slightly more general form of Equation 30 and a useful approximation for Equation 30 for spectral indices greater than or equal to two. The Rayleigh phase variance, $\sigma_{\phi R}^2$, was introduced in Reference 1 to distinguish between that portion of the phase variance that accounts for the Rayleigh component of the signal and that portion that represents a phase effects only random signal component. f_2' and f_1' as defined above apply to the delta layer. A reasonable extension for thick layers is available by applying a scale factor, H_m, to Equation 38 in Reference 1.

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$$\frac{1}{f_2'^2} = \frac{4\pi^2 H_m}{K^4 c^2} \int_0^{z_t} \frac{dz'}{z'^2} \int_0^{z'} \frac{dz}{z^2} \left[I_u^2(z) + I_v^2(z) + 2I_{uv}^2(z) \right] \tag{31}$$

where

$$I_u(z') = \int_0^{z'} dz \, B_n \frac{d\sigma_\phi^2}{dz} \left(\frac{2z^2 L_u^2}{L_x^2 L_y^2 - L_{xy}^2} \right)$$

$$I_v(z') = \int_0^{z'} dz \, B_n \frac{d\sigma_\phi^2}{dz} \left(\frac{2z^2 L_v^2}{L_x^2 L_y^2 - L_{xy}^2} \right)$$

$$I_{uv}(z') = \int_0^{z'} dz \, B_n \frac{d\sigma_\phi^2}{dz} \left(\frac{2z^2 L_{uv}}{L_x^2 L_y^2 - L_{xy}^2} \right)$$

$$H_m = \left[\int_0^{z_t} dz \, B_n \frac{d\sigma_\phi^2}{dz} \right]^{(4/m)-2}$$

and f_1' is redefined as

$$f_1' = f_2'/2^{\frac{1}{2}} \tag{32}$$

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Equations 29 through 32 provide a reasonable representation of frequency selective effects. For n greater than or equal to two, the results are quite accurate. For smaller n, the results are less accurate. The n equal to 3/2 case was examined in detail. The inaccuracy in Equations 29 through 32 were quite acceptable including the case where both the transmitter and the receiver were in the turbulent medium. The errors decrease as the layer thickness becomes comparable or smaller than the free space propagation distances.

The frequency selective bandwidth is still defined by:

$$f_0 = \left[\frac{\sigma_{\phi R}^2}{f_c^2} + \frac{1}{f_2^2} \right]^{-\frac{1}{2}} \tag{33}$$

4. THE PHASE EFFECTS ONLY SIGNAL COMPONENT

In the previous section, it was mentioned that a portion of the integrated phase spectrum determines a phase effects only signal component. This signal component can be represented as

$$U_\phi(t) = e^{i \left[\phi_L(t) + \int_{-\infty}^{\infty} df \, g(f) e^{i2\pi ft} \right]} \tag{34}$$

where $\phi_L(t)$ represents possible large scale trends in the phase and $g(f)$ is a zero mean normally distributed variable representing the random phase component. The total signal is the product of Equation 34 and the Rayleigh component. The variance of $g(f)$ is

$$\overline{g^*(f) g(f')} = \delta(f-f') P_\phi(f) \tag{35}$$

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where $g(-f)$ equals $g^*(f)$. An approximate expression for the phase power spectrum, $P_\phi(f)$, can be derived using the delta layer approximation.

Let

$$x = \frac{z}{z_t} V_x t \tag{36a}$$

$$y = \frac{z}{z_t} V_y t \tag{36b}$$

From Equation 12 and the delta layer approximation, the phase time correlation function is

$$R_\phi(t) = \frac{\sigma_\phi^2 \left(\frac{z_1}{z_t} \rho_v t\right)^{n-1}}{2^{n-1}\Gamma(n-1)} K_{n-1}\left(\frac{z_1}{z_t} \rho_v t\right) \tag{37a}$$

$$= \sigma_\phi^2 \left[1 - B_n\left(\frac{z_1}{z_t} \rho_v t\right)^m + \cdots \right] \tag{37b}$$

Also let

$$\tau_0 = \left[(\sigma_\phi^2 B_n)^{1/m} \frac{z_1}{z_t} \rho_v \right]^{-1} \tag{38}$$

The phase frequency spectrum is

$$P_\phi(f) = \frac{2\pi^{\frac{1}{2}}\Gamma(n-\frac{1}{2})(\sigma_\phi^2)^{\frac{m-2n+2}{m}} B_n^{\frac{2-2n}{m}} \tau_0}{\Gamma(n-1)\left[B_n^{-2/m}\sigma_\phi^{-4/m} + (2\pi f \tau_0)^2\right]^{n-\frac{1}{2}}} \tag{39}$$

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The phase power spectrum should be truncated at f_R where

$$2 \int_{f_R}^{\infty} P_\phi(f) df = \max(\sigma_{\phi R}^2, 0.025) \qquad (40)$$

The frequencies greater than f_R should be ignored because either they contribute to the Rayleigh signal component or they contribute little to the total integrated phase variance. From Equations 39 and 40

$$f_R = \frac{(\sigma_\phi^2)^{\frac{1}{2n-2} - \frac{1}{n}}}{2\pi\tau_0 \bar{b}_n^{1/n}} \left[\frac{\Gamma(n-\frac{1}{2})}{\pi^{\frac{1}{2}}(n-1)\Gamma(n-1)\max(\sigma_{\phi R}^2, 0.025)} \right]^{\frac{1}{2n-2}} \qquad (41)$$

In Equation 41, f_R is set to zero when $\sigma_{\phi R}^2$ equals σ_ϕ^2. Equation 39 will be equal to or greater than the actual phase spectral power. The actual phase spectrum for a given n will be shallower in slope than above because of geometric or physical non-global homogeneities in the index of refraction statistics. The two will agree at approximately f equal to f_R, thus the actual spectrum will fall below Equation 39 for frequencies less than f_R.

In applications where the structured index of refraction is due to ionization structure. The phase effects only phase spectrum is usually represented as a total electron content power spectrum.

$$P_{TEC}(f) = (f_c / r_0 c)^2 P_\phi(f) \qquad (42)$$

where r_0 is the classical electron radius.

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The development in this and the previous section assumed that n was a globally homogeneous quantity. Proposed algorithms for lifting this restriction are presented in Appendix C.

5.   SUMMARY.

The preceeding sections extended the results of Reference 1 to permit the prediction of the effects of structured index of refraction on electromagnetic waves to spectral indices of less than two. The phase effects only phase spectrum was developed, which along with the Rayleigh component constitute a complete statistical description of a scintillated signal. This work and that in Reference 1 form the basis of a signal structure specification for satellite C^3 applications proposed in Reference 12.

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APPENDIX A

CALCULATION OF THE MEAN SQUARE LOG AMPLITUDE FLUCTUATION AND THE RAYLEIGH PHASE VARIANCE

The equation for the mean square log amplitude fluctuation is

$$\overline{\chi^2} = \int_0^{z_t} dz \int_{-\infty}^{\infty} \frac{dK_x}{2\pi} \int_{-\infty}^{\infty} \frac{dK_y}{2\pi} \sin^2\left[\frac{(K_x^2+K_y^2)\,(z_t-z)z}{2Kz_t}\right] \frac{dP_\phi(K_x,K_y)}{dz} \quad \text{(A-1)}$$

where

$$\frac{dP_\phi(K_x,K_y)}{dz} = \frac{d\sigma_\phi^2}{dz} \frac{4\pi(L_x^2 L_y^2 - L_{xy}^2)^{1/2}(n-1)}{(1+L_x^2 K_x^2 + L_y^2 K_y^2 + 2L_{xy}K_x K_y)^n} \quad \text{(A-2)}$$

Equation A-1 using A-2 is not integrable but it can be reasonably approximated. Let

$$K_x = \kappa \cos \theta$$

$$K_y = \kappa \sin \theta$$

$$a^2 = \kappa^2 \frac{(z_t-z)z}{2Kz_t}$$

$$M(z) = \frac{(L_x^2+L_y^2)(z_t-z)z}{Kz_t(L_x^2 L_y^2 - L_{xy}^2)}$$

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$$\sin^2(a^2) = a^4, \quad a < c_2$$

$$= c_2^4, \quad a \geq c_2$$

The differential phase spectrum, in terms of a, is assumed constant for

$$a < c_1 M^{\frac{1}{2}}(z)$$

and power law for larger values. Finally let

$$a_1 = \min(c_2, c_1 M^{\frac{1}{2}}(z))$$

$$a_2 = \max(c_2, c_1 M^{\frac{1}{2}}(z))$$

The final approximation for $1.2 \leq n \leq 4$ is

$$\overline{\chi^2} = \frac{1}{2} \int_0^{z_t} dz \frac{d\sigma_\phi^2}{dz} (n-1)f(n) \left\{ \frac{8a_1^6}{3M(z)} + \frac{8c_2^4(a_2^2 - c_2^2)}{M(z)} \right.$$

$$\left. + I_0(z) + \frac{16c_1^{2n}c_2^4 M^{n-1}(z)a_2^{2-2n}}{2n-2} \right\} \qquad (A-3)$$

where

$$I_0(z) = \frac{16c_1^{2n}M^{n-1}(z)\left(c_2^{6-2n} - a_1^{6-2n}\right)}{6-2n}, \quad n \neq 3$$

$$I_0(z) = 16c_1^{2n}M^{n-1}(z)\ln\left(\frac{c_2}{a_1}\right), \quad n = 3$$

This range assumes that $\frac{d\sigma_\phi^2}{dz}$ is well defined for n less than or equal to 1 5.

With the following choices for c_1, c_2, and $f(n)$, the error in Equation A-3 is less than thirty percent.

$$c_1 = 0.5$$

$$c_2 = 0.84$$

$$f(n) = \left(\frac{f'(n)+M(z)/n}{1+M(z)}\right)\left\{1-\frac{n}{12}\exp\left[-\left(\frac{M(z)}{3} - \frac{1}{10M(z)} -1\right)^2\right]\right\}$$

$$f'(n) = 1.1-\max(0.0,0.5(n-2.4))$$

The correction factors apply for all M(z) and n between 1.2 and 4. They also apply for any value of L_x/L_y.

Equation A-3 is consistent and a bit more accurate than the $\overline{\chi^2}$ development in Reference 1. That result is still often preferable over its range of application because of its simplicity.

$$\overline{\chi^2} = \frac{1}{2}\int_0^{z_t} dz\,\frac{d\sigma_\phi^2}{dz}\left[\frac{M(z)}{1+M(z)}\right]^{n-1} \qquad (A-4)$$

The phase variance, for small M(z), is divisible into the Rayleigh phase variance and the phase effects only portion. The Rayleigh phase variance is defined as

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$$\sigma_{\phi R}^2 \cong \frac{1}{2} \int_0^{z_t} dz \, \frac{d\sigma_\phi^2}{dz} \, \frac{(n-1)M^{n-1}(z)}{4^{n-2}} \int_{a_c}^\infty \frac{da}{a^{2n-1}} \tag{A-5}$$

a_c is chosen using Equation A-3 to determine that portion of the integral that contributes $\overline{\chi_c^2}$ to $\overline{\chi^2}$. This condition is approximated by

$$\frac{1}{2} \int_0^{z_t} dz \, \frac{d\sigma_\phi^2}{dz} \, 16(n-1)f(n)c_1^{2n} M^{n-1}(z) \left[\int_{c_1 M^{\frac{1}{2}}(z)}^{\min(a_c, c_2)} da \, a^{5-2n} \right.$$
$$\left. + c_2^4 \int_{c_2}^{\max(a_c, c_2)} \frac{da}{a^{2n-1}} \right] + I = \overline{\chi_c^2} \tag{A-6}$$

where

$$I = \frac{1}{2} \int_0^{z_t} dz \, \frac{d\sigma_\phi^2}{dz} \, (n-1)f(n) \left[\frac{8a_1^6}{3M(z)} + \frac{8c_2^4(a_2^2 - c_2^2)}{M(z)} \right] \tag{A-7}$$

$\overline{\chi_c^2}$ is chosen as 0.1. The procedure to calculate $\sigma_{\phi R}^2$ is relatively straight forward. First, if $\overline{\chi^2} \leq 0.1$, then $\sigma_{\phi R}^2 = 0$. End procedure. If $I \geq \overline{\chi_c^2}$, then $\sigma_{\phi R}^2 = \sigma_\phi^2$. End procedure. Otherwise calculate

$$I_1 = \frac{1}{2} \int_0^{z_t} dz \, \frac{d\sigma_\phi^2}{dz} \, M^2(z) \tag{A-8a}$$

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$$I_2 = \frac{1}{2} \int_0^{z_t} dz \, \frac{d\sigma_\phi^2}{dz} M^{n-1}(z) \, , \, n \neq 3 \qquad \text{(A-8b)}$$

$$I_2 = \frac{1}{2} \int_0^{z_t} dz \, \frac{d\sigma_\phi^2}{dz} M^2(z) \ln\left(\frac{1}{M^2(z)}\right), \, n = 3 \qquad \text{(A-8c)}$$

Next calculate

$$I_3 = (I_2 c_2^{6-2n} - I_1 c_1^{6-2n})/(6-2n), \, n \neq 3 \qquad \text{(A-9a)}$$

$$I_3 = I_1 \ln\left(\frac{c_2}{c_1}\right) + I_2 \, , \, n = 3 \qquad \text{(A-9b)}$$

if $I_3 \leq 0$ and $\sigma_\phi^2/2 < \overline{x_c^2}$, then $\sigma_{\phi R}^2 = 0$. End procedure. If $I_3 \leq 0$ and $\sigma_\phi^2/2 \geq \overline{x_c^2}$, then $\sigma_{\phi R}^2 = \sigma_\phi^2$. End procedure. Otherwise calculate

$$\overline{x_s^2} = 16(n-1)f(n)c_1^{2n}I_3 + I \qquad \text{(A-10)}$$

If $\overline{x_c^2} - \overline{x_s^2} < 0$, then calculate

$$a_c = \left\{ (6-2n)\left[\frac{\overline{x_c^2} - \overline{x_s^2}}{16c_1^{2n}(n-1)f(n)I_2}\right] + c_2^{6-2n} \right\}^{\frac{1}{6-2n}}, \, n \neq 3 \qquad \text{(A-11a)}$$

$$a_c = c_2 \exp\left[\frac{\overline{x_c^2} - \overline{x_s^2}}{16c_1^{2n}(n-1)f(n)I_1}\right] \, , \, n = 3 \qquad \text{(A-11b)}$$

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If $\overline{\chi_c^2} - \overline{\chi_s^2} \geq 0$, then calculate

$$a_c = \left[c_2^{2-2n} + \frac{(2-2n)(\overline{\chi_c^2} - \overline{\chi_s^2})}{16 c_1^{2n} c_2^4 (n-1) f(n) I_2} \right]^{\frac{1}{2-2n}}, \quad n \neq 3 \qquad \text{(A-12a)}$$

$$a_c = \left[c_2^{2-2n} + \frac{(2-2n)(\overline{\chi_c^2} - \overline{\chi_s^2})}{16 c_1^{2n} c_2^4 (n-1) f(n) I_1} \right]^{\frac{1}{2-2n}}, \quad n = 3 \qquad \text{(A-12b)}$$

Finally

$$\sigma_{\phi R}^2 = \min \left[\frac{2 I_2}{4^{n-1} a_c^{2n-2}}, \sigma_\phi^2 \right], \quad n \neq 3 \qquad \text{(A-13a)}$$

$$\sigma_{\phi R}^2 = \min \left[\frac{2 I_1}{4^{n-1} a_c^{2n-2}}, \sigma_\phi^2 \right], \quad n = 3 \qquad \text{(A-13b)}$$

End procedure.

For $1.7 \leq n \leq 2.75$, the results from Reference 1 are still preferable because of their simplicity.

$$\sigma_{\phi R}^2 = \min \left\{ \sigma_\phi^2, \left[\frac{\overline{\chi_c^2}(6-2n)}{2n-2} \right]^{\frac{n-1}{n-3}} \left(\frac{(n-1)}{4^{n-2}} \overline{\chi^2} \right)^{\frac{2}{3-n}} \right\} \qquad \text{(A-14)}$$

Equation A-14 has comparable accuracy to the previous procedure.

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APPENDIX B
THE B_n COEFFICIENT

The B_n coefficient is defined by

$$B_n(\rho^2)^{m/2} \approx 1 - \frac{(\epsilon^2+\rho^2)^{\frac{n-1}{2}} K_{n-1}\left[(\epsilon^2+\rho^2)^{\frac{1}{2}}\right]}{c_n 2^{n-2}\Gamma(n-1)} \qquad (B-1)$$

where

$$3\epsilon \leq \rho \leq 0.1$$

ϵ = inner to outer scale ratio

m = minimum $(2, 2n-2)$

$$c_n = \frac{\epsilon^{n-1} K_{n-1}(\epsilon)}{2^{n-2}\Gamma(n-1)}$$

The inner scale has been introduced to insure physically meaningful results for n near two. Let us first examine n greater than or equal to two. First note that

$$\frac{(\epsilon^2+\rho^2)^{\frac{n-1}{2}} K_{n-1}\left[(\epsilon^2+\rho^2)^{\frac{1}{2}}\right]}{2^{n-2}\Gamma(n-1)} = \frac{2\Gamma(n-\frac{1}{2})}{\pi^{\frac{1}{2}}\Gamma(n-1)} \int_0^\infty \frac{\cos\left[(\epsilon^2+\rho^2)^{\frac{1}{2}}k\right]}{(1+k^2)^{n-\frac{1}{2}}} dk \qquad (B-2)$$

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By twice differentiating Equations B-1 and B-2, it is easy to show for any ρ less than about 0.1

$$B_n \approx \frac{\Gamma(n-\frac{1}{2})}{c_n \pi^{\frac{1}{2}} \Gamma(n-1)} \int_0^\infty \frac{k^2 \cos(\epsilon k)}{(1+k^2)^{n-\frac{1}{2}}} dk \qquad (B-3)$$

Equation B-3 can be approximated by

$$B_n \approx \frac{\Gamma(n-\frac{1}{2})}{c_n \pi^{\frac{1}{2}} \Gamma(n-1)} \left(\int_0^1 k^2 dk + \int_1^{1/\epsilon} k^{3-2n} dk \right) \frac{f(n)}{(n-1)} \qquad (B-4)$$

where $f(n)/(n-1)$ is a correction factor to correct for the approximation of the integral in Equation B-3. Now

$$B_n \approx \frac{\Gamma(n-\frac{1}{2})}{c_n \pi^{\frac{1}{2}} \Gamma(n-1)} \left[\frac{1}{3} + \frac{1}{2n-4} \left(1 - \epsilon^{2n-4} \right) \right] \frac{f(n)}{(n-1)}, \quad 2 \leq n \leq 4 \qquad (B-5a)$$

$$B_n \approx \frac{1}{2c_n} \left[\frac{1}{3} + \ln\left(\frac{1}{\epsilon}\right) \right] f(2), \quad n=2 \qquad (B-5b)$$

In the limit as ϵ becomes small

$$\underset{\epsilon \to 0}{\text{Limit}} \; B_n = \frac{1}{4} \frac{\Gamma(n-2)}{\Gamma(n-1)}, \quad n>2 \qquad (B-6)$$

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By using Equation B-6 and requiring continuity of B_n at n equal to two, the correction factor can be evaluated

$$f(n) \approx (n-1)^{-0.3}, \quad n \geq 2 \tag{B-7}$$

For n less than two, let us differentiate Equations B-1 and B-3 once. Then

$$B_n \approx \frac{\Gamma(n-\frac{1}{2})}{c_n \pi^{\frac{1}{2}}(n-1)\Gamma(n-1)} \int_0^\infty \frac{x \, \sin\left[(1+\epsilon^2/\rho^2)x\right]}{(\rho^2+x^2)^{n-\frac{1}{2}}} \, dx \tag{B-8}$$

where x equals $k\rho$. Equation B-8 can be approximated by

$$B_n \approx \frac{\Gamma(n-\frac{1}{2})}{c_n \pi^{\frac{1}{2}}\Gamma(n-1)}\left[\int_0^\rho x^2 dx + \int_\rho^1 x^{3-2n} dx + \int_1^\infty x^{2-2n} \sin(x) dz\right] \frac{f(n)}{(n-1)} \tag{B-9}$$

where it is assumed that ρ is greater than or equal to 5ϵ. Now

$$B_n \approx \frac{\Gamma(n-\frac{1}{2})}{c_n \pi^{\frac{1}{2}}\Gamma(n-1)} \left[\frac{\rho^3}{3} + \frac{1}{4-2n}\left(1-\rho^{4-2n}\right) + O(\frac{1}{2})\right] \frac{f(n)}{(n-1)} \tag{B-10}$$

As expected, Equation B-10 is relatively insensitive to ρ which can be thus thought of as a fit parameter. By choosing ρ equal to ϵ and fitting $f(n)$, Equation B-10 can be made continuous with Equation B-5. For small ϵ, it can be shown that

$$\text{Limit } B_n = \frac{\Gamma(n-\tfrac{1}{2})}{c_n \pi^{\tfrac{1}{2}} \Gamma(n-1)(n-1)} \, g(n), \quad n<2 \tag{B-11}$$
$$\underset{\varepsilon \to 0}{}$$

where
$$g(n) = \frac{\Gamma(4-2n)}{3-2n} \sin\left(\frac{\pi(3-2n)}{2}\right), \quad \frac{3}{2}<n<2$$

$$g(n) = \frac{\pi}{2}, \quad n=\frac{3}{2}$$

$$g(n) = \Gamma(3-2n) \sin\left(\frac{\pi(3-2n)}{2}\right), \quad 1<n<\frac{3}{2}$$

Using Equation B-11 and the continuity requirement at n equal to two, we find

$$f(n) = (n-1)^{-0.14} \tag{B-12}$$

The final approximation for B_n is

$$B_n = \frac{\Gamma(n-\tfrac{1}{2})}{c_n \pi^{\tfrac{1}{2}} \Gamma(n-1)} \left(\frac{1}{3} + h(\varepsilon)\right)\frac{f(n)}{(n-1)} \tag{B-13}$$

where
$$h(\varepsilon) = \frac{1}{|4-2n|}\left(1-\varepsilon^{|4-2n|}\right), \quad n\neq2$$

$$= \ln\left(\frac{1}{\varepsilon}\right), \quad n=2$$

$$f(n) = (n-1)^{-0.30}, \quad 2\leq n\leq4$$

$$f(n) = (n-1)^{-0.14}, \quad 1.2\leq n\leq2$$

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Equation B-13 is accurate to within twenty percent for the specified variable ranges.

In principle, the inner to outer scale size ratio is not an isotropic quantity. Fortunately, Equation 13 is insensitive to the precise value of ϵ, hence any reasonable estimate will do. To avoid underestimating effects, the maximum reasonable value of ϵ should be used. If this procedure is inadequate, then the anisotropy can be handled by defining B_n as the non-ϵ dependent portion of Equation B-13 and redefining the outer scales. For example, if n equals two, then the following definitions can be used.

$$B_n' = \frac{1}{2c_n} \tag{B-14}$$

$$L_r'^2 = L_r^2 / \left[\ln(L_r/1_r) + 1/3 \right] \tag{B-15a}$$

$$L_s'^2 = L_s^2 / \left[\ln(L_s/1_s) + 1/3 \right] \tag{B-15b}$$

$$L_t'^2 = L_t^2 / \left[\ln(L_t/1_t) + 1/3 \right] \tag{B-15c}$$

where 1_r, 1_s, and 1_t are the inner scales. These new quantities substitute for L_r, L_s, L_t, and B_n in all of the equations in this report.

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APPENDIX C

LOCALLY HOMOGENEOUS SPECTRAL INDICES

The developments in Reference 1 and the present paper have usually assumed that the fluctuation spectral index, or equivalently, n, is a globally homogeneous variable. This appendix will propose procedures to lift this limitation. It is assumed that the reader is familiar with the other portions of this paper including the preceding appendices.

Let us start with the mean square log amplitude fluctuation and the Rayleigh phase variance. Equation A-3 already applies for variable n. In preparation for calculation of the Rayleigh phase variance, Equation A-3 must be divided up into separate contributions from discrete values of n. Let

$$\overline{\chi^2} = \sum_i \overline{\chi_i^2} + \overline{\chi_s^2} \tag{C-1a}$$

$$\overline{\chi_s^2} = \frac{1}{2} \int_0^{z_t} dz \, \frac{d\sigma_\phi^2}{dz} \, (n-1) f(n) \left[\frac{8a_1^6}{3M(z)} + \frac{8c_2^4(a_2^2 - c_2^2)}{M(z)} \right] \tag{C-1b}$$

$$\overline{\chi_i^2} = I_{3i} + \frac{1}{2} \int_0^{z_t} dz \, \frac{d\sigma_\phi^2}{dz} \, h_i(n)(n-1) f(n) \frac{16c_1^{2n} c_2^4 M^{n-1}(z) a_2^{2-2n}}{2n-2} \tag{C-1c}$$

$$I_{3i} = \frac{1}{2}\int_0^{z_t} dz \; \frac{d\sigma_\phi^2}{dz} \; h_i(n)(n-1)f(n)I_0(z) \qquad (C\text{-}1d)$$

where
$$h_i(n) = 1, \quad |n-n_i| \le \Delta n_i/2$$

$$= 0, \quad |n-n_i| > \Delta n_i/2$$

If the range of n includes three, then for some i

$$n_i - \Delta n_i/2 = 3 \text{ and/or } n_{i-1} + \Delta n_{i-1}/2 = 3$$

The remaining quantities are defined in Appendix A. If $\overline{\chi^2} \le \overline{\chi_c^2}$, then $\sigma_{\phi R}^2 = 0$. Otherwise calculate

$$I_{1i} = \frac{1}{2}\int_0^{z_t} dz \; \frac{d\sigma_\phi^2}{dz} \; h_i(n)(n-1)f(n)M^2(z) \qquad (C\text{-}2a)$$

$$I_{2i} = \frac{1}{2}\int_0^{z_t} dz \; \frac{d\sigma_\phi^2}{dz} \; h_i(n)(n-1)f(n)M^{n-1}(z) \qquad (C\text{-}2b)$$

Let
$$S_i = 1, \quad I_{3i} > 0$$

$$= 0, \quad I_{3i} \le 0$$

$$I_3 = \sum_i S_i I_{3i}$$

If $I_3 \leq 0$ and $\dfrac{\sigma_\phi^2}{2} \leq \overline{\chi_c^2}$, then $\sigma_{\phi R}^2 = 0$. End. If $I_3 \leq 0$ and $\dfrac{\sigma_\phi^2}{2} > \overline{\chi_c^2}$, then $\sigma_{\phi R}^2 = \sigma_\phi^2$. End. Let

$$\overline{\chi_c^2}' = \overline{\chi_c^2} - \overline{\chi_s^2} - \sum_i \overline{\chi_i^2}(1-S_i) \tag{C-3}$$

If $\overline{\chi_c^2}' \leq 0$, then $\sigma_{\phi R}^2 = \sigma_\phi^2$. End procedure. Otherwise solve for a_c in the following equation.

$$8\sum_i c_1^{2n_i} S_i \left\{ \max\left[\left(\frac{I_{2i} \min^{6-2n_i}(a_c,c_2) - I_{1i} c_1^{6-2n_i}}{3-n_i} \right), 0 \right] \right.$$
$$\left. + c_2^4 I_{2i} \left[\frac{\max^{2-2n_i}(a_c,c_2) - c_2^{2-2n_i}}{1-n_i} \right] \right\} = \overline{\chi_c^2}' \tag{C-4}$$

Finally

$$\sigma_{\phi R}^2 = 2\sum_i \frac{S_i I_{2i} a_c^{2-2i}}{4^{n_i-1}} \tag{C-5}$$

The next parameter is the perpendicular signal decorrelation time, $\tau_\perp$. First, let

$$a_i = \int_0^{z_t} dz\, \frac{d\sigma_\phi^2}{dz}\, h(n_i) B_n \left(\frac{z^2 \rho_v^2}{z_t^2} \right)^{m/2} \tag{C-6}$$

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$\tau_{\perp}$ then satisfies the following equation.

$$\sum_i a_i \tau_{\perp}^{m_i} = -\ln\left[e^{-1} + e^{-\sigma_\phi^2}(1-e^{-1})\right] \tag{C-7}$$

where m_i = minimum $(2, 2n_i - 2)$. We also define the effective time index.

$$n_\tau \equiv \frac{\sum_i n_i a_i \tau^{m_i}}{\sum_i a_i \tau^{m_i}} \tag{C-8}$$

Next we calculate

$$C_{ui} = \int_0^{z_t} dz \frac{d\sigma_\phi^2}{dz} h(n_i) B_n \left[\left(\frac{z}{z_t}\right)^2 \frac{L_u^2}{L_x^2 L_y^2 - L_{xy}^2}\right]^{m/2} \tag{C-9a}$$

$$C_{vi} = \int_0^{z_t} dz \frac{d\sigma_\phi^2}{dz} h(n_i) B_n \left[\left(\frac{z}{z_t}\right)^2 \frac{L_v^2}{L_x^2 L_y^2 - L_{xy}^2}\right]^{m/2} \tag{C-9b}$$

$$C'_{uvi} = \int_0^{z_t} dz \frac{d\sigma_\phi^2}{dz} h(n_i) B_n \left[\left(\frac{z}{z_t}\right)^2 \frac{|L_{uv}|}{L_x^2 L_y^2 - L_{xy}^2}\right]^{m/2} \text{sign}(L_{uv}) \tag{C-9c}$$

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Now calculate a_u and a_v where

$$\sum_i C_{ui} a_u^{m_i} = 1 \qquad\qquad\text{(C-10a)}$$

$$\sum_i C_{vi} a_{vi}^{m_i} = 1 \qquad\qquad\text{(C-10b)}$$

The angular index is defined as

$$n_\theta \equiv \frac{\sum_i n_i (C_{ui} a_u^{m_i} + C_{vi} a_v^{m_i})}{\sum_i (C_{ui} a_u^{m_i} + C_{vi} a_v^{m_i})} \qquad\qquad\text{(C-11)}$$

Also

$$m_\theta = \text{minimum } (2, 2n_\theta - 2) \qquad\qquad\text{(C-12)}$$

Now

$$C_u = \left(\sum_i C_{ui} \; a_u^{m_i - m_\theta} \right)^{2/m_\theta} \qquad\qquad\text{(C-13a)}$$

$$C_v = \left(\sum_i C_{vi} \; a_v^{m_i - m_\theta} \right)^{2/m_\theta} \qquad\qquad\text{(C-13b)}$$

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$$C_{uv} = |\Sigma|^{2/m_\theta} \text{ sign } (\Sigma) \qquad (C-13c)$$

$$\Sigma = \sum_i C'_{uvi} (a_u a_v)^{\frac{m_i - m_\theta}{2}} \qquad (C-13d)$$

The above coefficients are then used in Equation 19 with m_θ substituting for m. Thus

$$G(z_t, u, v) \simeq e^{-(C_u u^2 + C_v v^2 - 2C_{uv} uv)^{m_\theta/2}} \qquad (C-14)$$

Equations 21 through 26 then follow. Equation 30 remains a reasonable choice for the generalized power spectrum where the index, m, is redefined as

$$m = \text{minimum } (m_\theta, m_\tau) \qquad (C-15)$$

where

$$m_\tau = \text{minimum } (2, 2n_\tau - 2) \qquad (C-16)$$

Equation C-15 imposes a worst case functional form for the generalized power spectrum with regards to both the frequency and delay. For situations where frequency selectivity is not significant it is best to use m_τ for m to avoid overestimating the fade rate effects. Conversely, if frequency selective effects dominate, m_θ is probably the best choice. If both effects are operative, then Equation C-15 is advised

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so as to not underestimate the effects. In any event, non-globally homogeneous spectral indices imply some compromise in the fidelity of the generalized power spectrum form so some conservatism is good.

Equation 31 through 33 remain reasonable with m_θ replacing m in Equation 31. Similarly Equations 39 and 41 can be used with n_τ and m_τ replacing n and m_c respectively.

The choices for n_i and Δn_i depend on the range of n and the required resolution. If n varies continuously, then several values of n are necessary with the finest resolution for small values of n. Another possibility is where n has a number of discrete values corresponding to different physical processes in different regions. In this case, Δn_i is chosen as some arbitrarily small value. The formalism does not allow n_i equal to three, but any choice arbitrarily close to three is adequate.

This completes the formalism for locally homogeneous spectral indices. Unfortunately, this formalism is significantly more complex and costly to implement over that for a globally spectral index. This formalism also increases the degree of approximation particularly with regards to Equation C-14, the functional form of the generalized power spectrum, $f_2^!$, and the functional form of the phase effects only phase power spectrum. These results still provide, however, reasonable estimates of the signal structures and their resulting effects.

APPENDIX D
THE TWO FREQUENCY MUTUAL CORRELATION FUNCTION
AND ANTENNA EFFECTS

To derive the various results of this appendix, the most general equation for the two frequency mutual correlation function is required. Let that function be

$$G(z,x,y,X,Y,\eta) = \overline{U^*(z,\overline{r}_1,K_1) \, U(z,\overline{r}_2,K_2)} \qquad \text{(D-1)}$$

where the orignial propagating signal is expressed as

$$E(z,\overline{r},K) = \frac{U(z,\overline{r},K)e^{iK(|r|^2+z^2)^{\frac{1}{2}}}}{z} \qquad \text{(D-2)}$$

and

$$\eta = \frac{K_2 - K_1}{K_2}$$

$$\hat{x}x + \hat{y}x = \overline{r}_2 - \overline{r}_1$$

$$X\hat{x} + Y\hat{y} = \frac{\overline{r}_2 + \overline{r}_1}{2}$$

The equation for G is

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$$\frac{dG}{dz} = -\frac{i}{2K_2}\left(\frac{\eta}{1-\eta}\right)\left[\frac{d^2G}{dx^2} + \frac{d^2G}{dy^2} + \frac{1}{4}\left(\frac{d^2G}{dX^2} + \frac{d^2G}{dY^2}\right)\right] - \frac{x}{z}\frac{dG}{dx} - \frac{y}{z}\frac{dG}{dy}$$

$$-\frac{X}{z}\frac{dG}{dX} - \frac{Y}{z}\frac{dG}{dY} + \frac{i(2-\eta)}{2K_2(1-\eta)}\left[\frac{d^2G}{dxdX} + \frac{d^2G}{dydY}\right]$$

$$-\left\{\frac{1}{2}\left[1 + \frac{1}{(1-\eta)^2}\right]\frac{d\sigma_\phi^2}{dz} - \frac{1}{1-\eta}\frac{dR_\phi(x,y,X,Y)}{dz}\right\}G \qquad (D-3)$$

Equation 17 results from Equation D-3 by neglecting variation of $R_\phi(x,y,X,Y)$ with respect to X and Y and by assuming no initial dependence of G on X and Y.

Equation D-3 can be used along with the delta layer approximation and considerations from antenna theory to get the generalized power spectrum including antenna effects. Let us assume that the receiving and transmitting antennas are circular aperatures with gains of G and G', respectively. In terms of the antenna aperature diameters

$$G = \left(\frac{\pi D}{\lambda}\right)^2 \qquad (D-4a)$$

$$G' = \left(\frac{\pi D'}{\lambda}\right)^2 \qquad (D-4b)$$

where λ is the propagation wavelength.

Assuming isotropic fluctuations about the propagation line of sight and $\sigma_\phi^2 > 1$, the generalized power spectrum is

$$\Gamma_2(f, \tau, \theta_x, \theta_y, \theta_x', \theta_y') = (2\pi)^{\frac{1}{2}} f_2' \left(\frac{\sigma_\phi R^{f}_2}{f_c}\right)^{-1} \frac{\lambda^2 GG'}{(4\pi z_t)^2}$$

$$\int_{-\infty}^{\infty} dR \int_{-\infty}^{\infty} dM \int_{-\infty}^{\infty} d(\Delta t) e^{-\left(M^2 + \frac{\Delta t^2}{\tau_0^2}\right)^{m/2}} e^{i2\pi RM} e^{i2\pi f \Delta t}$$

$$e^{-\frac{1}{2}\left(\frac{\sigma_\phi R^{f}_2}{f_c}\right)^{-2} \left[2\pi f_2' \tau - (\pi \tau_0 f)^2 - (\pi R)^2\right]^2} \frac{4J_1^2(s)}{s^2} \frac{4J_1^2(s')}{s'^2} \quad (D\text{-}5)$$

where

$$s^2 = G\left[(2^{\frac{1}{2}}\pi \tau_0 f \sigma_\theta + \theta_x)^2 + (2^{\frac{1}{2}}\pi R \sigma_\theta + \theta_y)^2\right]$$

$$s'^2 = G'\left[(2^{\frac{1}{2}}\pi \tau_0 f \sigma_\theta' + \theta_x')^2 + (2^{\frac{1}{2}}\pi R \sigma_\theta' + \theta_y')^2\right]$$

θ_x, θ_y = receiving antenna pointing error

θ_x', θ_y' = transmitting antenna pointing error

$J_1(s)$ = Bessel function

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$\sigma_{\phi R}^2$ = Rayleigh phase variance

$\sigma_{\theta}'^2$ = energy angle of arrival variance at transmitter

m = minimum of two and spectral index

For one dimensional structure transverse to the propagation line of sight, insert $\delta(M) \delta(R)$ at the end of Equation D-5 and integrate over R and M. The gain factors and the free space propagation loss have been left in Equation E-5. Usually these terms are handled separately.

By dropping these terms, assuming a spectral index greater than or equal to two, zeroing the pointing errors, and approximating the Bessel functions terms by a gaussian function, the following generalized power spectrum can be calculated.

$$\Gamma_2(f,\tau) = \frac{2^{3/4}\pi^{\frac{1}{4}}f_2''\tau_0'}{A}\left(\frac{f_c}{f_2''\sigma_{\phi R}}\right) e^{-(\pi\tau_0'f)^2} e^{-\frac{1}{2}\left[(\pi\tau_0'f)^2 - 2\pi f_2''\tau\right]^2}\left(\frac{f_c}{f_2''\sigma_{\phi R}}\right)^2$$

$$F\left\{\frac{1}{2^{\frac{1}{4}}}\left(\frac{f_2''\sigma_{\phi R}}{f_c}\right)\left[1 + \left((\pi\tau_0'f)^2 - 2\pi f_2''\tau\right)\left(\frac{f_c}{f_2''\sigma_{\phi R}}\right)^2\right]\right\} \qquad \text{(D-6)}$$

where

$$F(z) = \int_{-\infty}^{\infty} dx \, e^{-x^4 - 2zx^2}$$

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$$\Gamma_1(f,\tau) = \frac{2^{1/2}\pi\tau_0' f_1''}{A^{1/2}}\left(\frac{f_c}{f_1''\sigma_\phi R}\right) e^{-(\pi\tau_0' f)^2}\; e^{-\frac{1}{2}\left[(\pi\tau_0' f)-2\pi f_1'\tau\right]^2}\left(\frac{f_c}{f_1''\sigma_\phi R}\right)^2 \qquad \text{D-7}$$

where
$$A = \left(1 + \frac{G\sigma_\theta^2}{2.0} + \frac{G'\sigma_\theta^{2'}}{2.0}\right)$$

$$\tau_0' = \tau_0\, A^{1/2}$$

$$f_n'' = f_n'\, A$$

and the Bessel function terms were approximated by

$$\frac{4J_1^2(s)}{s^2} \simeq e^{-\frac{s^2}{4}} \qquad (D-8)$$

The functional form of the power spectrums is identical to that in Reference 1. Thus, the results in Reference 1 and 12 can be used after scaling τ_0 and f_0 and accounting for the power loss. The scaling of τ_0 and f_0 is generally beneficial to systems. Unfortunately, the power loss generally negates much of the gain unless there is a large margin.

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